

Concept of Equivalent Diameter for Heat and Fluid Flow in Porous Media

A. Nakayama, F. Kuwahara, and Y. Sano

Dept. of Mechanical Engineering, Shizuoka University, 3-5-1 Johoku, Hamamatsu, 432-8561 Japan

DOI 10.1002/aic.11092

Published online January 29, 2007 in Wiley InterScience (www.interscience.wiley.com).

Keywords: porous media, thermal dispersion, heat transfer coefficient, packed bed

Introduction

In this note, we shall propose the concept of the equivalent diameter and the Reynolds number based on it, which can handle both hydrodynamic and heat-transfer aspects associated with fluid flow, and heat transfer in porous media. This concept enables us to extract a series of useful correlations for porous media, such as the friction factor, thermal dispersion and interfacial heat transfer coefficient, from existing correlations available for tube flows and cross flows over banks of cylinders.

Prior to the discussion leading to the equivalent diameter, here we revisit the concept of the hydraulic diameter often introduced in conjunction with the Carman-Kozeny theory, to estimate the permeability of a fluid-saturated porous medium (See for example,¹). The hydraulic diameter d_h may best be illustrated in consideration of a bundle of capillary tubes as shown in Figure 1, in which, Hagen-Poiseuille flow may develop.

For each tube, the following equation should hold

$$-\frac{d\langle p \rangle^f}{dx} = 32\mu_f \frac{\langle u \rangle^f}{d_t^2} = \frac{\mu_f}{(d_t^2 \varepsilon / 32)} \langle u \rangle \quad (1)$$

where d_t is the capillary tube diameter and

$$\langle \phi \rangle^f = \frac{1}{V_f} \int_{V_f} \phi \, dV \quad (2)$$

in general denotes the intrinsic averaged value of ϕ over the volume space V_f occupied by the fluid within the local control volume V , where the subscript f stands for the fluid phase. Another volume averaging may be used as

$$\langle \phi \rangle = \frac{1}{V} \int_V \phi \, dV \quad (3)$$

The obvious relationship between the Darcian velocity $\langle u \rangle$, and the pore velocity $\langle u \rangle^f$, namely, $\langle u \rangle = \varepsilon \langle u \rangle^f$ (where $\varepsilon = V_f/V$), is used in Eq. 1. The hydraulic diameter concept (often called the Carman-Kozeny theory) runs as

$$K \equiv \frac{\mu_f \langle u \rangle}{-\frac{d\langle p \rangle^f}{dx}} = \frac{\varepsilon d_h^2}{16k_k} = \frac{\varepsilon^3}{k_k(1-\varepsilon)^2 A_0^2} \quad (4)$$

where K is the permeability and the hydraulic diameter is given by

$$d_h = \frac{4\varepsilon}{(1-\varepsilon)A_0} \quad (5)$$

Obviously, the hydraulic diameter d_h corresponds with the tube diameter d_t with the Kozeny constant $k_k = 2$ for the case of a bundle of circular tubes. In the last expression in Eq. 4, the volumetric area based on the solid structure A_0 is introduced such that $A_0 = 4/d_{cir}$ for the case of circular cylinder particles of diameter d_{cir} , and $A_0 = 6/d_p$ for the case of spherical particles of diameter d_p . For the packed beds, the Kozeny constant k_k is approximated as some value around 4 to 5. Such two typical expressions for K are given by

$$K = \frac{\varepsilon^3}{150(1-\varepsilon)^2 d_p^2} \quad (\text{Ergun}) \quad (6a)$$

$$K = \frac{\varepsilon^3}{180(1-\varepsilon)^2 d_p^2} \quad (\text{Carman - Kozeny}) \quad (6b)$$

The concept of hydraulic diameter provides a relationship between the pressure gradient and the porosity for different configurations such as cylindrical and spherical particles. Its discussion, however, is limited to the hydrodynamic aspects.

Equivalent Diameter

Let us define the equivalent diameter as follows

$$d_{eq} \equiv \sqrt{\left. 32 \frac{\mu \langle u \rangle^f}{-\frac{d\langle p \rangle^f}{dx}} \right|_{\langle u \rangle \rightarrow 0}} = \sqrt{32 \frac{K}{\varepsilon}} \quad (7)$$

Correspondence concerning this article should be addressed to A. Nakayama at tmanaka@ipc.shizuoka.ac.jp.

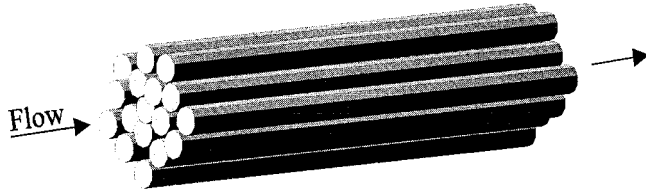


Figure 1. Flow through a bundle of capillary tubes.

The advantage of using the equivalent diameter will be clear, as we find it all we need to establish the correlations, whereas the use of the hydraulic diameter requires another geometrical parameter, namely, the Kozeny constant k_k .

For example, we have $d_{eq} = d_t$ from Eq. 1 for the circular tube flow and

$$d_{eq} = \sqrt{\frac{32}{150} \frac{\varepsilon}{1-\varepsilon}} d_p \quad : \text{Spheres} \quad (8)$$

from Eq. 6a for the packed beds. Furthermore, Kuwahara et al.² and Nakayama et al.³ conducted a series of numerical experiments for various arrangements of cubes, circular and square cylinders and established the following functional relationships

$$K = \frac{\varepsilon^3 d_{cube}^2}{152(1-\varepsilon)^2} \quad \text{hence } d_{eq} = \sqrt{\frac{32}{152} \frac{\varepsilon}{1-\varepsilon}} d_{cube} \quad : \text{Cubes} \quad (9)$$

$$K = \frac{\varepsilon^3 d_{cir}^2}{144(1-\varepsilon)^2} \quad \text{hence } d_{eq} = \sqrt{\frac{32}{144} \frac{\varepsilon}{1-\varepsilon}} d_{cir} \quad : \text{Circular cylinders in cross flows} \quad (10)$$

$$K = \frac{\varepsilon^3 d_{sq}^2}{120(1-\varepsilon)^2} \quad \text{hence } d_{eq} = \sqrt{\frac{32}{120} \frac{\varepsilon}{1-\varepsilon}} d_{sq} \quad : \text{Square cylinders in cross flows} \quad (11)$$

Thus, all two- and three-dimensional (3-D) obstacles lead to similar expressions for the permeability and hence for the equivalent diameter. It is particularly interesting to note that, for a fixed porosity, the permeability of the packed bed is almost the same as that of the bank of circular cylinders under $d_{cir} = d_p$. The equivalent diameters for these configurations are listed in Table 1 for a further discussion. We may roughly estimate the equivalent diameter as $d_{eq} = 0.5\varepsilon d_\phi / (1-\varepsilon)$ where the subscript ϕ corresponds to p , $cube$, cir or sq .

Friction Factor

Ergun's empirical equation for the packed beds⁴ is given by

$$-\frac{d\langle p \rangle^f}{dx} = \frac{150(1-\varepsilon)^2}{\varepsilon^2 d_p^2} \mu_f \langle u \rangle^f + 1.75 \frac{1-\varepsilon}{\varepsilon} \rho_f \frac{(\langle u \rangle^f)^2}{d_p} \quad : \text{Packed beds} \quad (12)$$

which may be rewritten in terms of the friction factor λ_{eq} using the equivalent diameter as

$$\lambda_{eq} \equiv \left[-\frac{d\langle p \rangle^f}{dx} \right] / \left[\frac{\rho_f (\langle u \rangle^f)^2}{2d_{eq}} \right] = \frac{64}{Re_{d_{eq}}} + 1.62 \quad (13)$$

where

$$Re_{d_{eq}} \equiv \frac{\langle u \rangle^f d_{eq}}{\nu_f} = \frac{0.462 \langle u \rangle d_p}{1-\varepsilon} \quad (14)$$

is the Reynolds number based on the intrinsic velocity and the equivalent diameter. Kuwahara et al.⁵ carried out the DSN and LES study to investigate the cross flows through periodic arrays of square cylinders in a staggered arrangement. Wide ranges of porosity and cross-flow angle were covered in this numerical study to establish the following expression

$$-\frac{d\langle p \rangle^f}{dx} = \frac{120(1-\varepsilon)^2}{\varepsilon^2 d_{sq}^2} \mu_f \langle u \rangle^f + 2.0 \frac{1-\varepsilon}{\varepsilon} \rho_f \frac{(\langle u \rangle^f)^2}{d_{sq}} \quad : \text{Square cylinders in cross flows} \quad (15)$$

Or in terms of the friction factor as

$$\lambda_{eq} \equiv \left[-\frac{d\langle p \rangle^f}{dx} \right] / \left[\frac{\rho_f (\langle u \rangle^f)^2}{2d_{eq}} \right] = \frac{64}{Re_{d_{eq}}} + 2.07 \quad (16)$$

where

$$Re_{d_{eq}} \equiv \frac{\langle u \rangle^f d_{eq}}{\nu_f} = \frac{0.516 \langle u \rangle d_{sq}}{1-\varepsilon} \quad (17)$$

In Figure 2, these two equations for the equivalent friction factor are plotted against the equivalent Reynolds number. The comparison of the Ergun empirical formula and the LES results obtained for the arrays of square cylinders suggests that the Ergun formula, when based on the equivalent diameter, is a such a universal law that can be used to estimate the pressure drops over the banks of two-dimensional obstacles, such as circular and square cylinders. This suggests that, under the concept of equivalent diameter, we can translate the results obtained for the packed bed into those for other porous configurations. In what follows, we shall exploit this equivalent diameter concept to capture the heat transfer characteristics in porous media, using correlations available for tube flows and cross flows over banks of cylinders.

Table 1. Equivalent Diameter

Configuration	Circular Tubes	Spheres	Cubes	Circular Cylinders	Square Cylinders
d_{eq}	d_t	$d_{eq} = 0.462 \frac{\varepsilon}{1-\varepsilon} d_p$	$d_{eq} = 0.459 \frac{\varepsilon}{1-\varepsilon} d_{cube}$	$d_{eq} = 0.471 \frac{\varepsilon}{1-\varepsilon} d_{cir}$	$d_{eq} = 0.516 \frac{\varepsilon}{1-\varepsilon} d_{sq}$

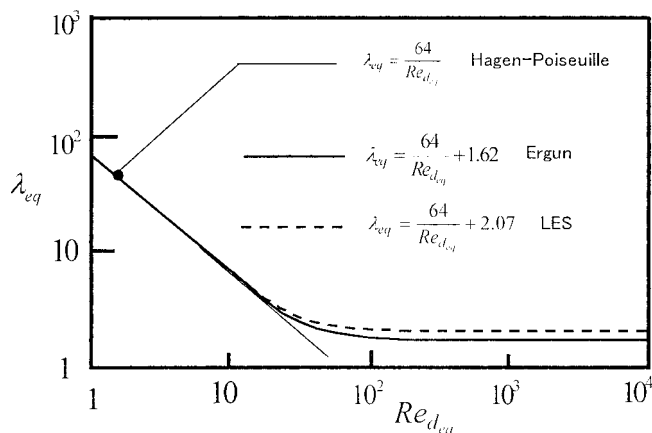


Figure 2. Equivalent friction factor.

Thermal Dispersion

In addition to the molecular thermal diffusion, there is significant mechanical dispersion in heat and fluid flow in a fluid-saturated porous medium, as a result of hydrodynamic mixing of the fluid particles passing through pores. This thermal dispersion causes additional heat transfer, which arises further complications in dealing with transport processes in fluid saturated porous media. In order to illustrate the mechanism of thermal dispersion, Taylor^{6,7} considered the rate of spreading of the heat (or mass) content caused by the radial velocity nonuniformity of the fluid flowing inside a circular tube and obtained the following analytical expression for the axial thermal conductivity $(k_{dis})_{ax}$ due to the dispersion in the case of high Peclet number

$$\frac{(k_{dis})_{ax}}{k_f} = 5.03 \left(\frac{u_\tau d_t}{\alpha_f} \right) \quad : \text{Taylor} \quad (18)$$

where k_f and α_f are the thermal conductivity and diffusivity of the fluid, respectively, and u_τ is the friction velocity. In order to translate the foregoing results for the packed bed, we use Ergun's empirical Eq. 13 for high Reynolds number as

$$u_\tau^2 = -\frac{d\langle p \rangle^f}{dx} \frac{d_{eq}}{4\rho_f} = \frac{\lambda_{eq}}{8} (\langle u \rangle^f)^2 \simeq \frac{1.62}{8} (\langle u \rangle^f)^2 = 0.203 (\langle u \rangle^f)^2 \quad (19)$$

Thus, we obtain the approximate expression for the axial dispersion coefficient as follows

$$\frac{(k_{dis})_{ax}}{k_f} = 5.03 \left(\frac{u_\tau d_t}{\alpha_f} \right) = 2.26 \left(\frac{\langle u \rangle^f d_t}{\alpha_f} \right) \quad (20)$$

We shall exploit the equivalent diameter concept, replacing $\langle u \rangle^f d_t / \alpha_f$ in the foregoing equation by the Reynolds number as given by Eq. 14. Thus, we have

$$\begin{aligned} \frac{k_{eff}}{k_f} &= \varepsilon \left(1 + \frac{(k_{dis})_{ax}}{k_f} \right) \cong \varepsilon \frac{(k_{dis})_{ax}}{k_f} = 2.26\varepsilon \left(\frac{0.462 \langle u \rangle d_p}{1 - \varepsilon \alpha_f} \right) \\ &\simeq 0.7 \left(\frac{\langle u \rangle d_p}{\alpha_f} \right) \quad (21) \end{aligned}$$

where the porosity for the packed bed is assumed to be $\varepsilon = 0.4$. Moreover, $\langle u \rangle d_p / \nu_f$, commonly used for a packed bed, is the particle Reynolds number based on the particle diameter d_p and Darcian velocity $\langle u \rangle = \varepsilon \langle u \rangle^f$. Also, note that k_{eff} is the effective axial thermal conductivity of the fluid. The foregoing expression for the packed beds is presented in Figure 3 along with the empirical formula proposed by Yagi et al.⁸ for the packed bed in a high Peclet number range, namely

$$\frac{k_{eff}}{k_f} = 0.5 \left(\frac{\langle u \rangle d_p}{\alpha_f} \right) \quad : \text{Yagi et al. (empirical)} \quad (22)$$

The present expression (Eq. 21) for the axial dispersion coefficient based on the equivalent diameter concept closely follows the empirical formula established by Yagi and his coworkers. In fact, they reported a tremendous scatter in the experimental data. The solid line based on the present formula passes through the scatter of these data.

Interfacial Heat Transfer Coefficient

Another important modeling parameter for non-local thermal equilibrium heat transfer within porous media may be the interfacial heat-transfer coefficient h_{sf} , which is required for describing the heat transfer from the fluid and solid interface A_{sf} , as follows

$$\int_{A_{sf}} k_f \nabla T \cdot d\vec{A} \equiv A_{sf} h_{sf} (\langle T \rangle^s - \langle T \rangle^f) \quad (23)$$

Zhukauskas^{9,10} carried out a series of exhaustive experiments to investigate the heat transfer from banks of circular cylinders and proposed the following correlations for the staggered arrangements

$$\begin{aligned} \frac{hd_{cir}}{k_f} &= 0.35 Pr^{0.36} \left(\frac{u_{max} d_{cir}}{\nu_f} \right)^{0.6} \\ &\text{for } 10^3 < \frac{u_{max} d_{cir}}{\nu_f} < 2 \times 10^5 \quad (24a) \end{aligned}$$

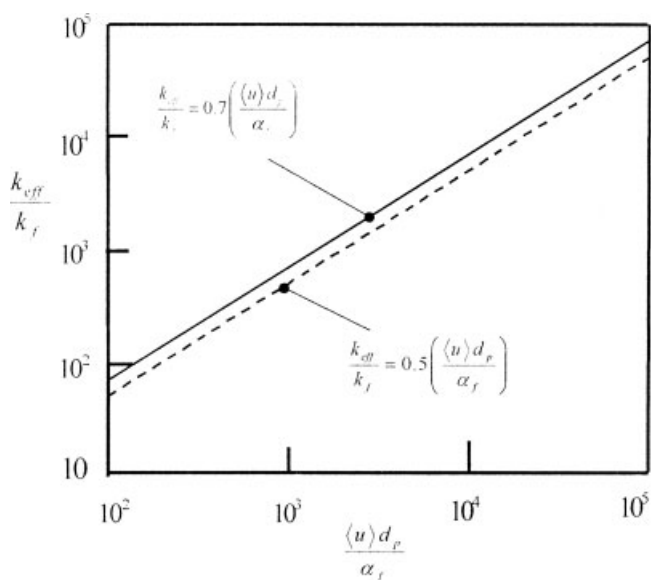


Figure 3. Axial thermal-dispersion coefficient.

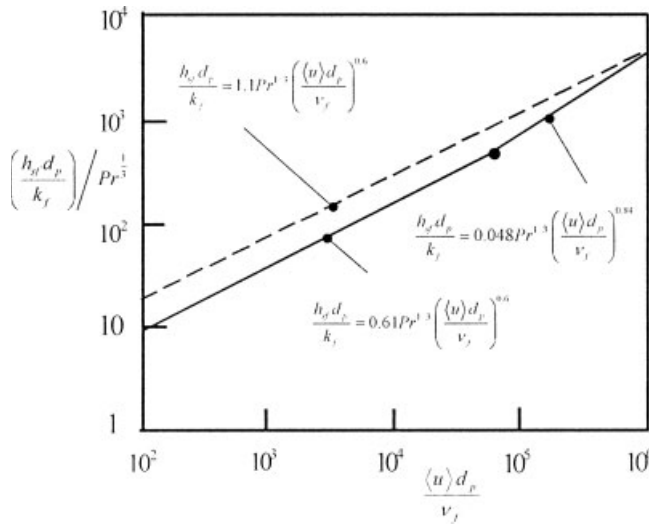


Figure 4. Interfacial heat-transfer coefficient.

and

$$\frac{hd_{cir}}{k_f} = 0.022 Pr^{0.36} \left(\frac{u_{max} d_{cir}}{v_f} \right)^{0.84}$$

for $2 \times 10^5 < \frac{u_{max} d_{cir}}{v_f} < 2 \times 10^6$ (24b)

where u_{max} is the average velocity at the minimum cross-sectional area. We note $d_{cir} = \sqrt{144/150} d_p = 0.98 d_p$ for the same value of d_{eq} , and exploit the equivalent diameter concept to obtain the corresponding correlation for the packed beds, replacing $u_{max} d_{cir}/v_f$ in the foregoing equation by $0.98 \langle u \rangle d_p / \varepsilon v_f$ as follows

$$\frac{h_{sf} d_p}{k_f} = \frac{0.35 Pr^{0.36}}{0.98} \left(\frac{0.98 \langle u \rangle d_p}{\varepsilon v_f} \right)^{0.6} \simeq 0.61 Pr^{1/3} \left(\frac{\langle u \rangle d_p}{v_f} \right)^{0.6}$$

for $4 \times 10^2 < \frac{\langle u \rangle d_p}{v_f} < 8 \times 10^4$ (25a)

and

$$\frac{h_{sf} d_p}{k_f} = \frac{0.022}{0.98} Pr^{0.36} \left(\frac{0.98 \langle u \rangle d_p}{\varepsilon v_f} \right)^{0.84} \simeq 0.048 Pr^{1/3} \left(\frac{\langle u \rangle d_p}{v_f} \right)^{0.84}$$

for $8 \times 10^4 < \frac{\langle u \rangle d_p}{v_f} < 8 \times 10^5$ (25b)

where the porosity for a packed bed is assumed to be $\varepsilon = 0.4$. These correlations based on the equivalent diameter concept are plotted in Figure 4 to compare with the empirical formula proposed by Wakao and Kaguei¹¹, which, for the high Reynolds number range, reduces to

$$\frac{h_{sf} d_p}{k_f} = 1.1 Pr^{1/3} \left(\frac{\langle u \rangle d_p}{v_f} \right)^{0.6} \quad \text{Wakao and Kaguei (empirical)}$$

(26)

The present expressions Eqs. 25a and 25b for the interfacial heat-transfer coefficient obtained using the equivalent diameter

concept, consistently follow the empirical formula (Eq. 26), which Wakao and Kaguei established, compiling a number of available experimental data. The underestimation by the present correlations should not be taken seriously, in view of the scatter of the experimental data (see Figure 4.3 in¹¹).

Conclusions

The concept of the equivalent diameter was proposed for extracting a set of useful correlations for porous media, such as the friction factor, thermal dispersion and interfacial heat-transfer coefficient, from the correlations available for tube flows and cross flows over banks of cylinders. It has been shown that the Ergun formula, when based on the equivalent diameter, can give a universal law for the friction factor in a variety of porous media. Moreover, the expressions established using the equivalent diameter concept for the thermal dispersion and interfacial heat-transfer coefficient in the packed beds, were found to agree well with the existing empirical formulas. Thus, the concept of equivalent diameter has proven to be sound.

Notation

d_{cir}	= diameter of circular cylinder
d_{cube}	= size of cube
d_{eq}	= equivalent diameter
d_h	= hydraulic diameter
d_p	= diameter of sphere
d_{sq}	= size of square cylinder
d_t	= diameter of tube
h_{sf}	= interfacial convective heat-transfer coefficient
K	= permeability tensor, directional permeability
Pr	= Prandtl number
u	= velocity
p	= pressure
k_k	= Kozeny constant
k_f	= thermal conductivity of fluid
Re_{deg}	= Reynolds number based on d_{eq} and the intrinsic velocity
V	= elementary representative volume
x	= axial coordinate
α_f	= fluid thermal diffusivity
ε	= porosity
λ_{eq}	= equivalent friction factor
λ_f	= friction factor
ν_f	= kinematic viscosity
ρ_f	= fluid density
μ_f	= fluid viscosity

Subscripts and superscripts

dis	= dispersion
eq	= equivalent
f	= fluid
s	= solid
sf	= interface

Special symbols

$\langle \rangle$	= volume-average
$\langle \rangle^{f,s}$	= intrinsic average

Literature Cited

1. Kaviani M. *Principles of Heat Transfer in Porous Media*. 1st ed., Springer-Verlag, New York; 1991:30–31.
2. Kuwahara F, Nakayama A, Koyama H. Numerical Modelling of Heat and Fluid Flow in a Porous Medium. *Proc. 10th Int. Heat Transfer Conference*, 1994;5:309–314.
3. Nakayama A, Kuwahara F, Kawamura Y, Koyama H. Three-Dimensional Numerical Simulation of Flow Through a Microscopic Porous Structure, *Proc. 4th ASME/JSME Thermal Engineering Conference*. 1995;3:313–318.
4. Ergun S. Flow Through Packed Columns, *Chem Eng Prog* 1952; 48:89–94.
5. Kuwahara F, Yamane T, Nakayama A. Large Eddy Simulation of Turbulent Flow in Porous Media, *Int. Communications in Heat and Mass Transfer*. 2006;33:411–418.
6. Taylor GI. Dispersion of solute matter in solvent flowing slowly through a tube. *Proc. Roy Soc London*. 1953;15:1787–1806.
7. Taylor GI. The dispersion of matter in turbulent flow in a pipe. *Proc Roy Soc London* 1954;A223:446–468.
8. Yagi S, Kunii D, Wakao N. Studies on axial effective thermal conductivities in packed beds, *AIChE Journal*, 1960;6:543–546.
9. Zhukauskas A. Konvektivnyi Perenos v Teploobmennikakh (Convective Transfer in Heat Exchangers), Nauka, Moscow; 1982;472.
10. Zhukauskas A. Heat transfer from tubes in crossflow, *Advances in Heat Transfer*. 1987;18:87–159.
11. Wakao N, Kaguei S. Heat and Mass Transfer in Packed Beds, Gordon and Breach Science Publishers, London, 1982;156–204.

Manuscript received Mar. 31, 2006, and final revision received Dec. 11, 2006.